UNIVERSITY OF CAMBRIDGE

Institute for Advanced Study

The effects of stochastic forces on planetary systems and Saturn's rings

Hanno Rein @AMNH New York, October 2010

1. Multi-planetary systems

   - Standard Model
   - Turbulent disc

2. Saturn's rings

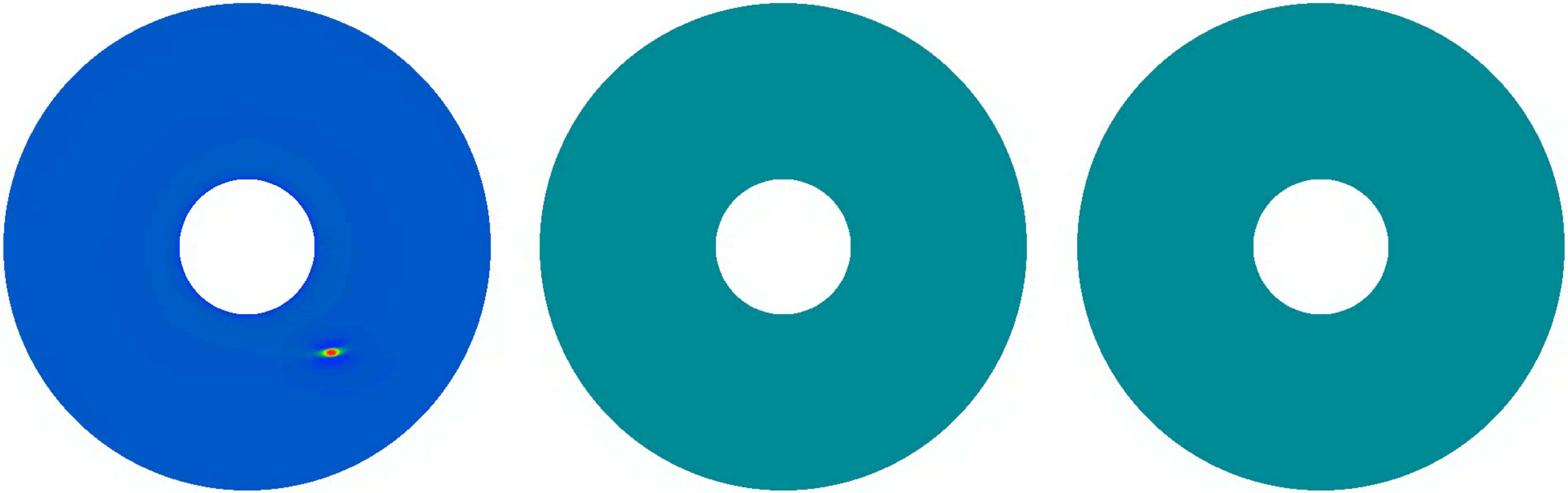
Exoplanets

1. Multi-planetary systems

- Standard Model
- Turbulent disc

2. Saturn’s rings

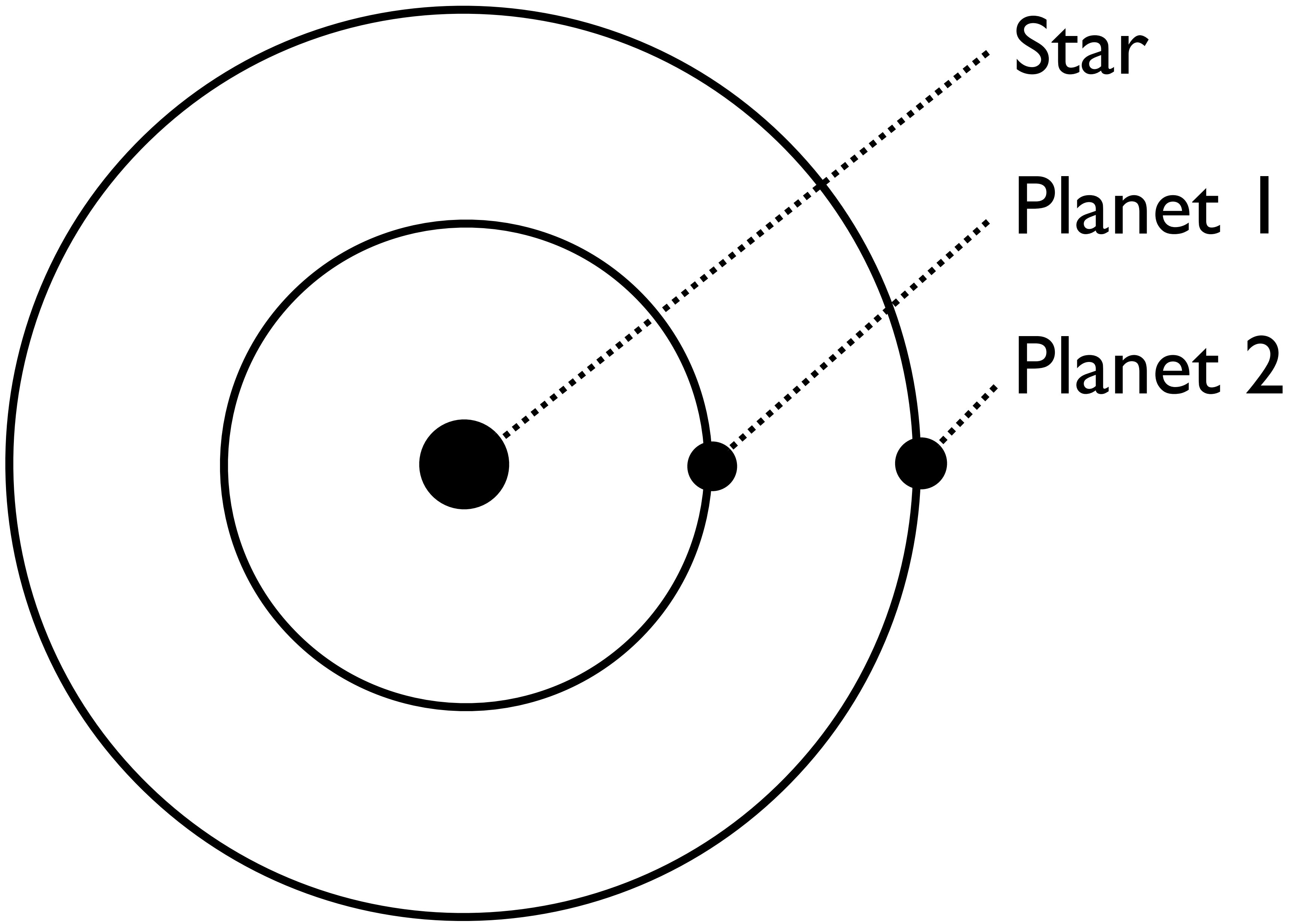
Planet migration



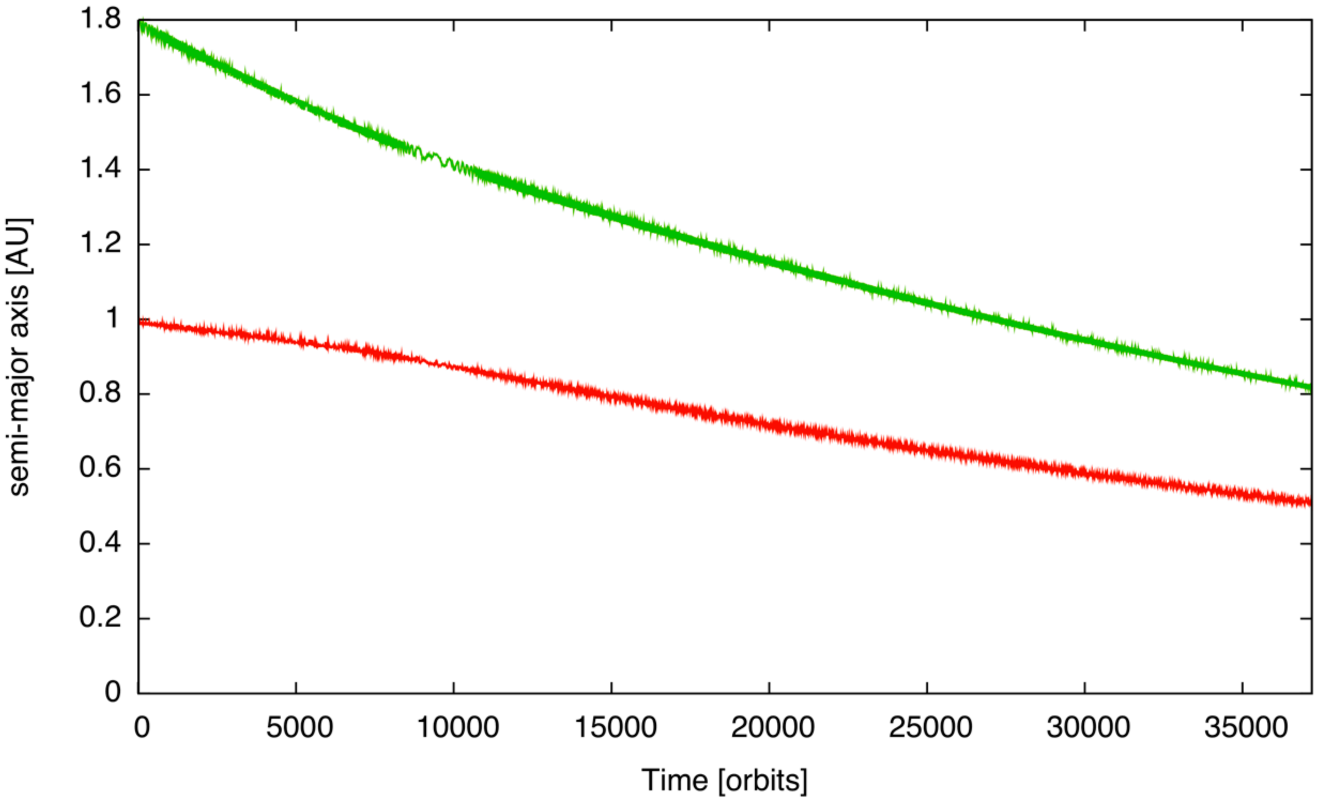
All planets migrate

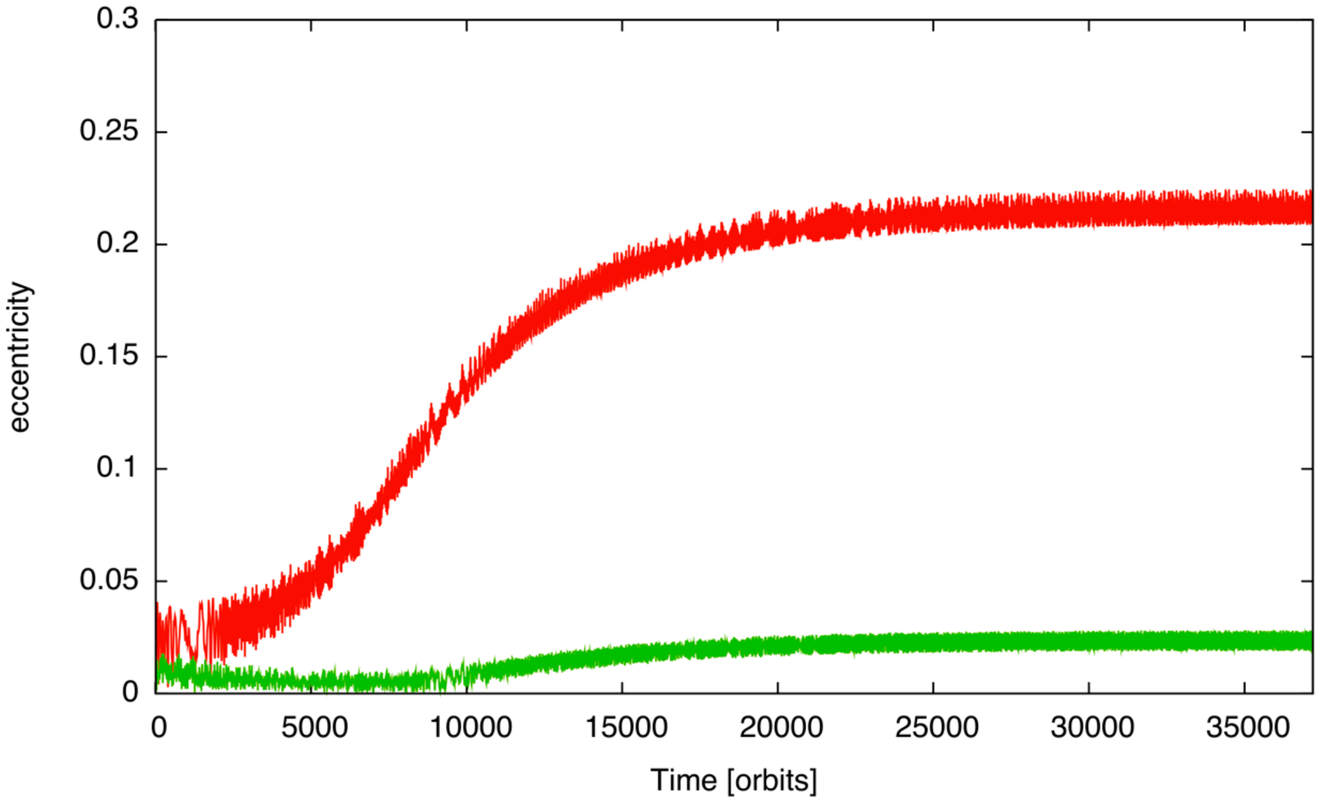
Migration rates vary

Mean motion resonance



Resonant planetary systems

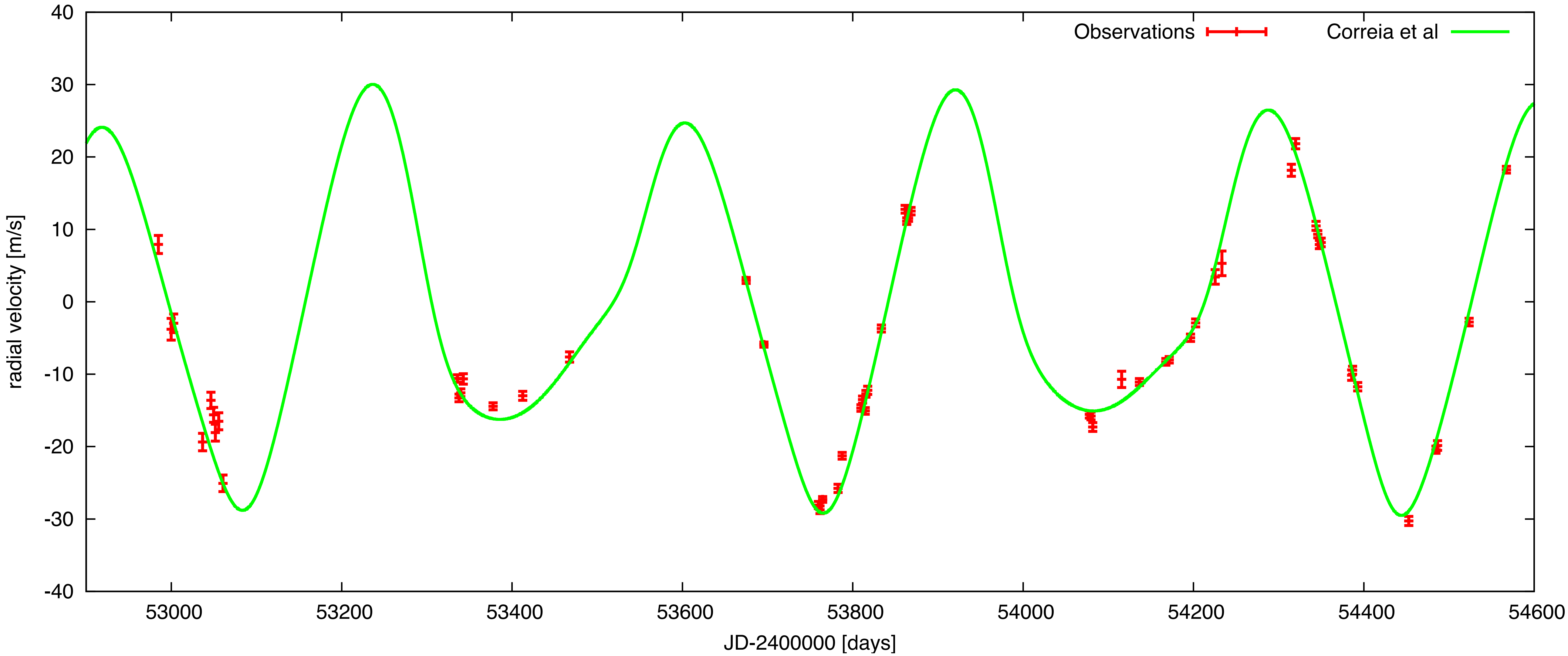




- Convergent migration leads to resonant capture
- N-body or hydrodynamical simulation
- Successful in explaining a range of system: GJ876, 55 Cancri, HD73526, ...

Lee & Peale 2002, Kley & Nelson 2008, Sandor et al 2007

... HD45364



Correia et al 2009

Formation scenario

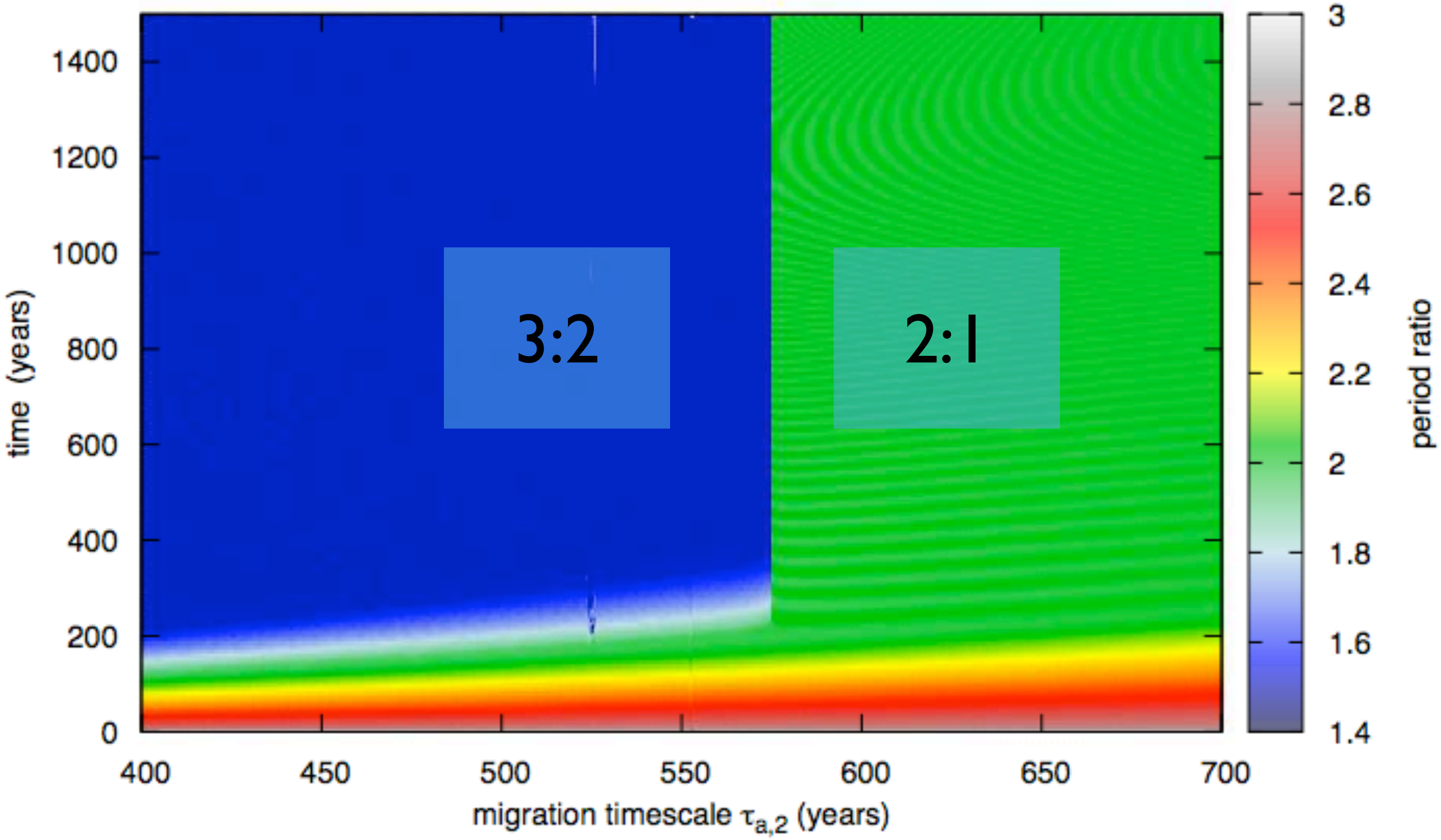
Have:
- Two planets
- Disc

→

Want:
- 3:2 resonance

- Infinite number of resonances
- How to choose?
- Initial positions
- Migration speed is crucial
- Resonance width and libration period define critical migration rate

N-Body simulations



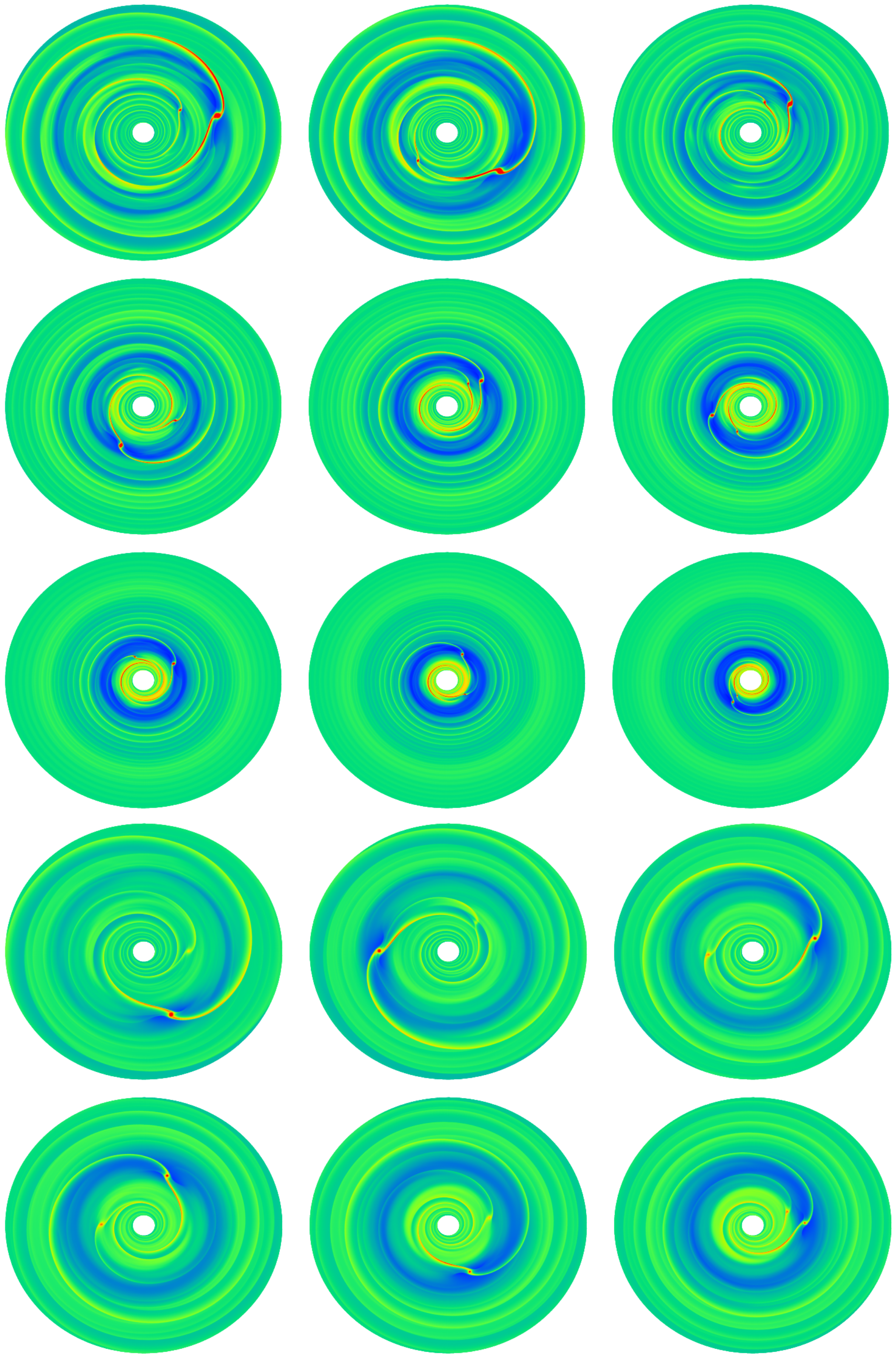
Hydro simulations

Massive disc (5 times MMSN)

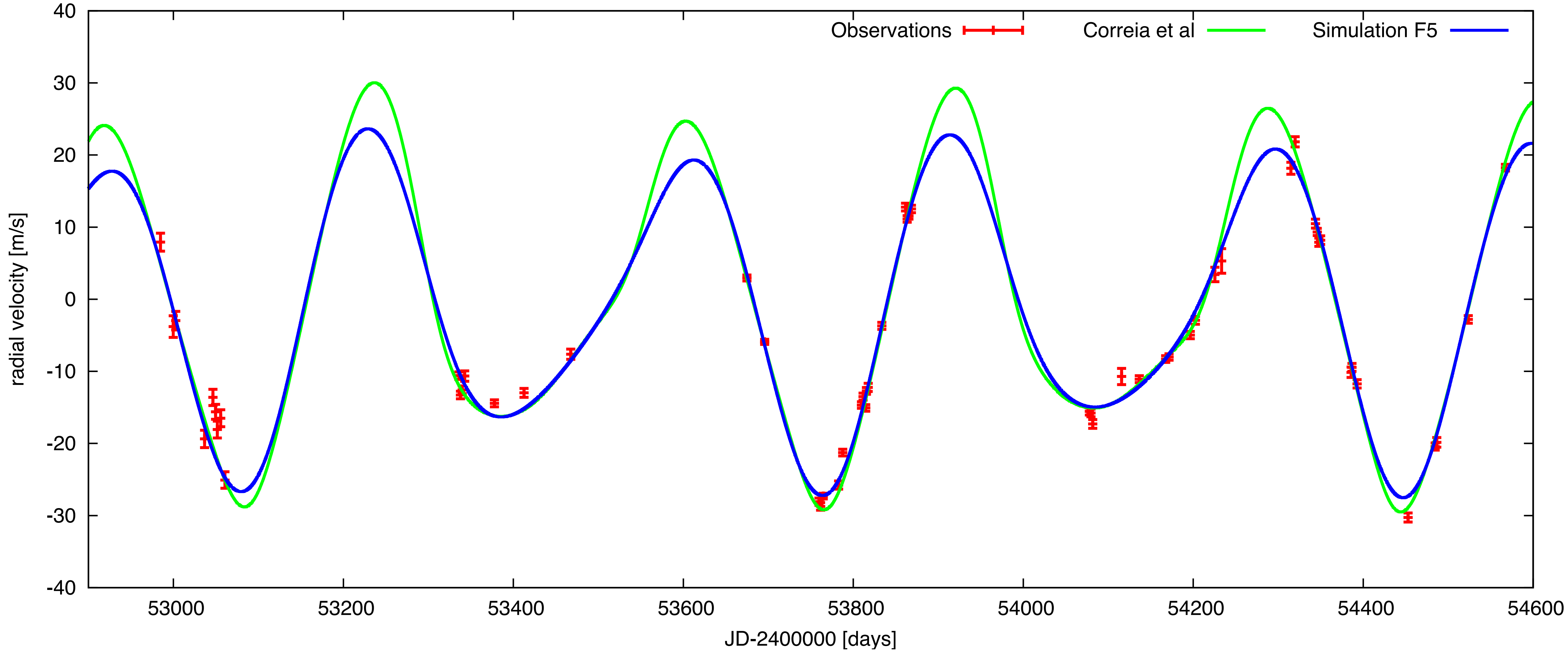
- Short, rapid Type III migration
- Passage of 2:1 resonance
- Capture into 3:2 resonance

Large scale-height (0.07)

- Slow Type I migration once in resonance
- Resonance is stable
- Consistent with radiation hydrodynamics



Formation scenario leads to improved 'fit'



Parameter	Unit	Correia et al. (2009) b	c	Simulation **F5** b	c
$M \sin i$	$[\mathrm{M_{Jup}}]$	0.1872	0.6579	0.1872	0.6579
M_*	$[M_\odot]$	0.82		0.82	
a	[AU]	0.6813	0.8972	0.6804	0.8994
e		0.17 ± 0.02	0.097 ± 0.012	0.036	0.017
λ	[deg]	105.8 ± 1.4	269.5 ± 0.6	352.5	153.9
ϖ^a	[deg]	162.6 ± 6.3	7.4 ± 4.3	87.9	292.2
$\sqrt{\chi^2}$		2.79		2.76^b (3.51)	
Date	[JD]	2453500		2453500	

Rein, Papaloizou & Kley 2010

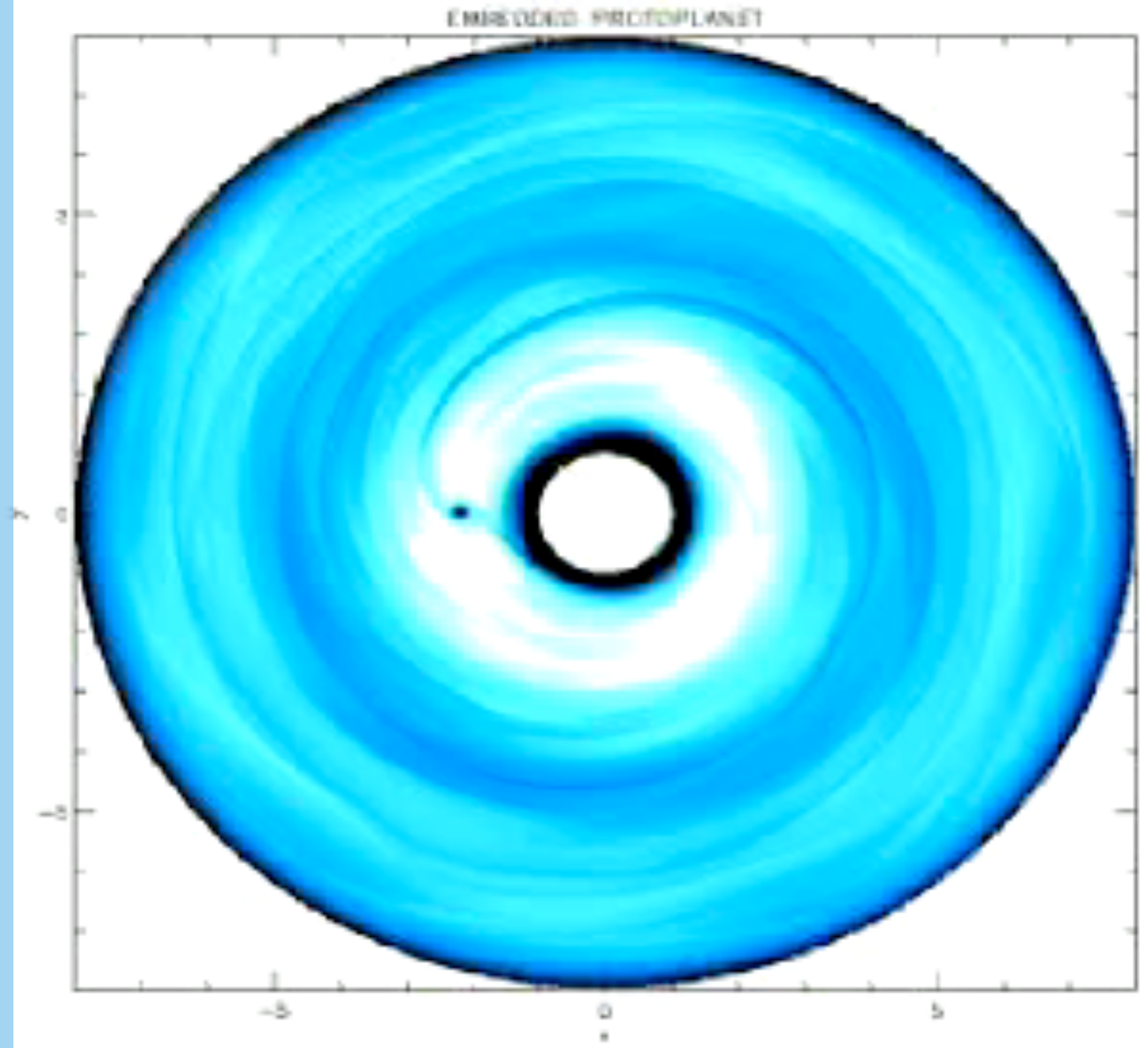
1. Multi-planetary systems

- Standard Model
- Turbulent disc

2. Saturn’s rings

Turbulent disc

- Angular momentum transport
- Magnetorotational instability (MRI)
- Density perturbations interact gravitationally with planets
- Stochastic forces lead to random walk
- Large uncertainties



Animation from Nelson & Papaloizou 2004
Random forces measured by Laughlin et al. 2004, Nelson 2005, Oischi et al. 2007

Level of abstraction

Analytic model

Describing evolution in a statistical manner

Adams 2008, Rein & Papaloizou 2009

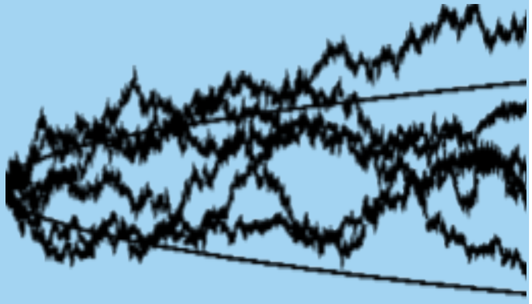
$$\Delta a = \sqrt{4\frac{Dt}{n^2}}$$

$$\Delta e = \sqrt{2.5\frac{\gamma Dt}{n^2 a^2}}$$

N-body simulations

Generating random forces, integrating planets directly

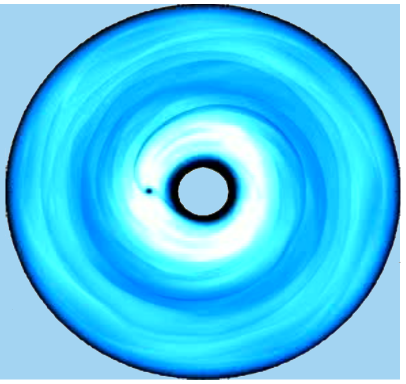
Rein & Papaloizou 2009, Baruteau & Lin 2010



Full 3D MHD simulations

Stratification, dead zones, non-ideal MHD,...

Nelson & Papaloizou 2004, Rein et al. 2013



Hamiltonian formalism

$$H \;\rightarrow\; H - m\,(F_x\,x + F_y\,y) = H - m\,(\mathbf{r}\cdot\mathbf{F})$$

$$\begin{aligned}
\dot{G}_F &= m\left(\frac{\partial}{\partial\lambda} + \frac{\partial}{\partial\varpi}\right)(\mathbf{r}\cdot\mathbf{F}) = m\,(\mathbf{r}\times\mathbf{F})\cdot\hat{\mathbf{e}}_z \\
\dot{E}_F &= mn\frac{\partial}{\partial\lambda}(\mathbf{r}\cdot\mathbf{F}) = m\,(\mathbf{v}\cdot\mathbf{F}) \\
\dot{\varpi}_F &= \frac{\sqrt{(1-e^2)}}{na\,e}\left[F_\theta\left(1+\frac{1}{1-e^2}\frac{r}{a}\right)\sin f - F_r\cos f\right] \\
\dot{\lambda}_F &= -m\left(\frac{\partial}{\partial L} + n\frac{\partial}{\partial E}\right)(\mathbf{r}\cdot\mathbf{F}) \\
&= \left(1-\sqrt{1-e^2}\right)\dot{\varpi}_F + \frac{2an}{\mathcal{G}M}\,(\mathbf{r}\cdot\mathbf{F}),
\end{aligned}$$

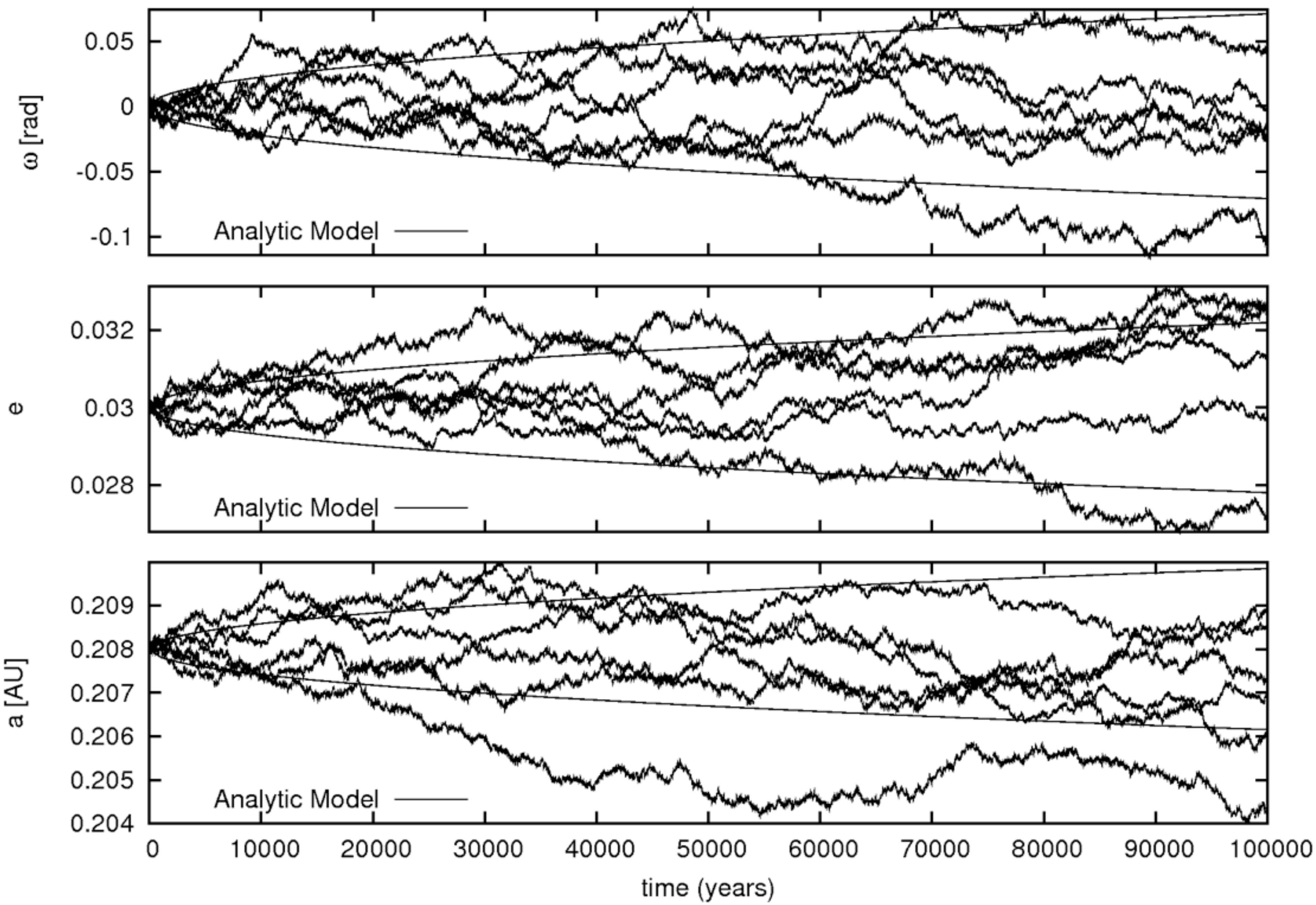
Analytic growth rates

$$
\begin{aligned}
(\Delta a)^2 &= 4\frac{Dt}{n^2} \\
(\Delta e)^2 &= 2.5\frac{\gamma Dt}{n^2 a^2} \\
(\Delta \varpi)^2 &= \frac{2.5}{e^2}\frac{\gamma Dt}{n^2 a^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{(\Delta \phi_1)^2}{(p+1)^2} &= \frac{9\gamma_f}{a_1^2 \omega_{lf}^2}\, D\, t \\
(\Delta(\Delta \varpi))^2 &= \frac{5\gamma_s}{4 a_1^2 n_1^2 e_1^2}\, D\, t
\end{aligned}
$$

Rein & Papaloizou 2009

N-body simulations



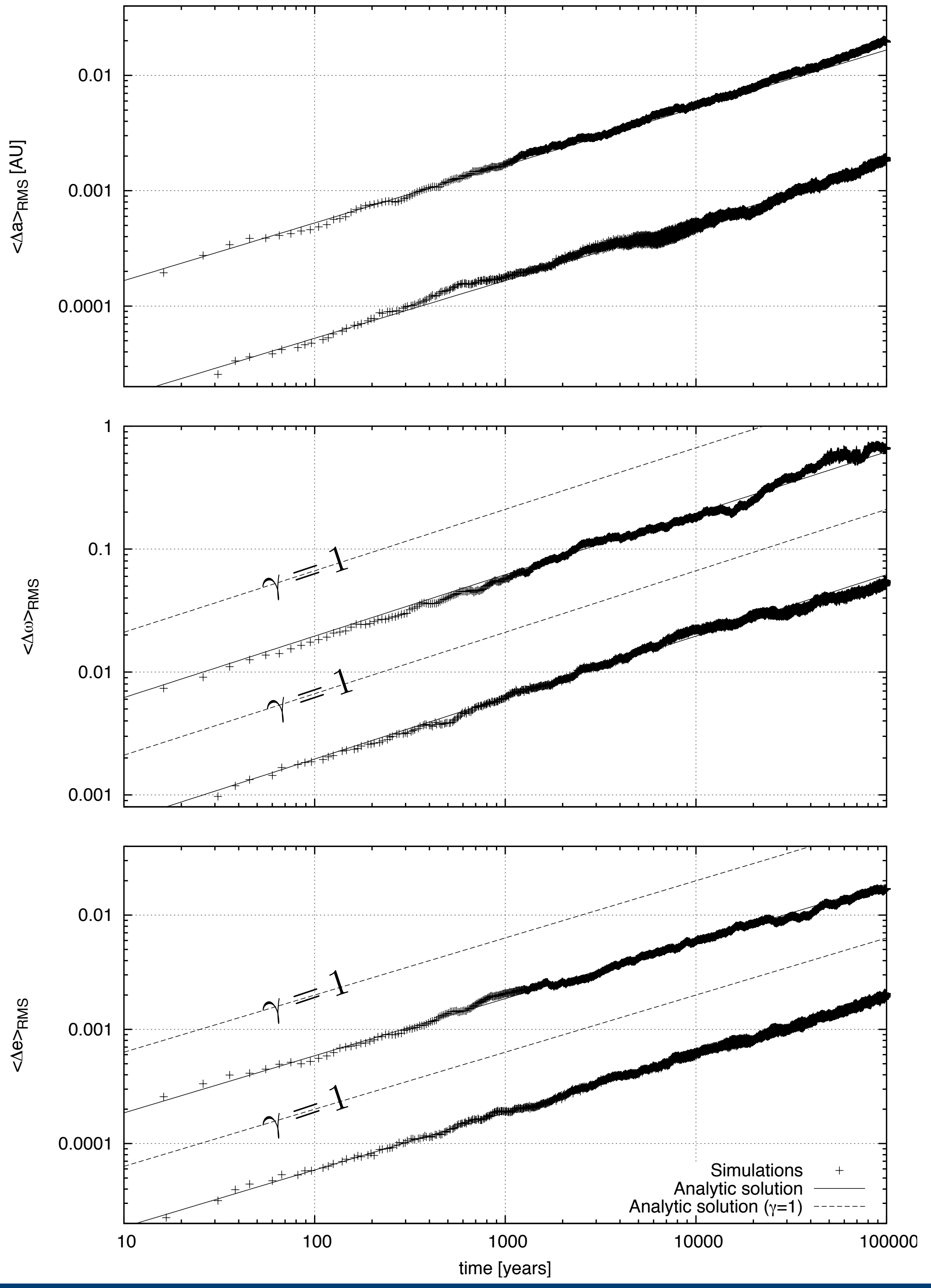
Rein & Papaloizou 2009

Correction factors are important

$$(\Delta a)^2 = 4\frac{Dt}{n^2}$$

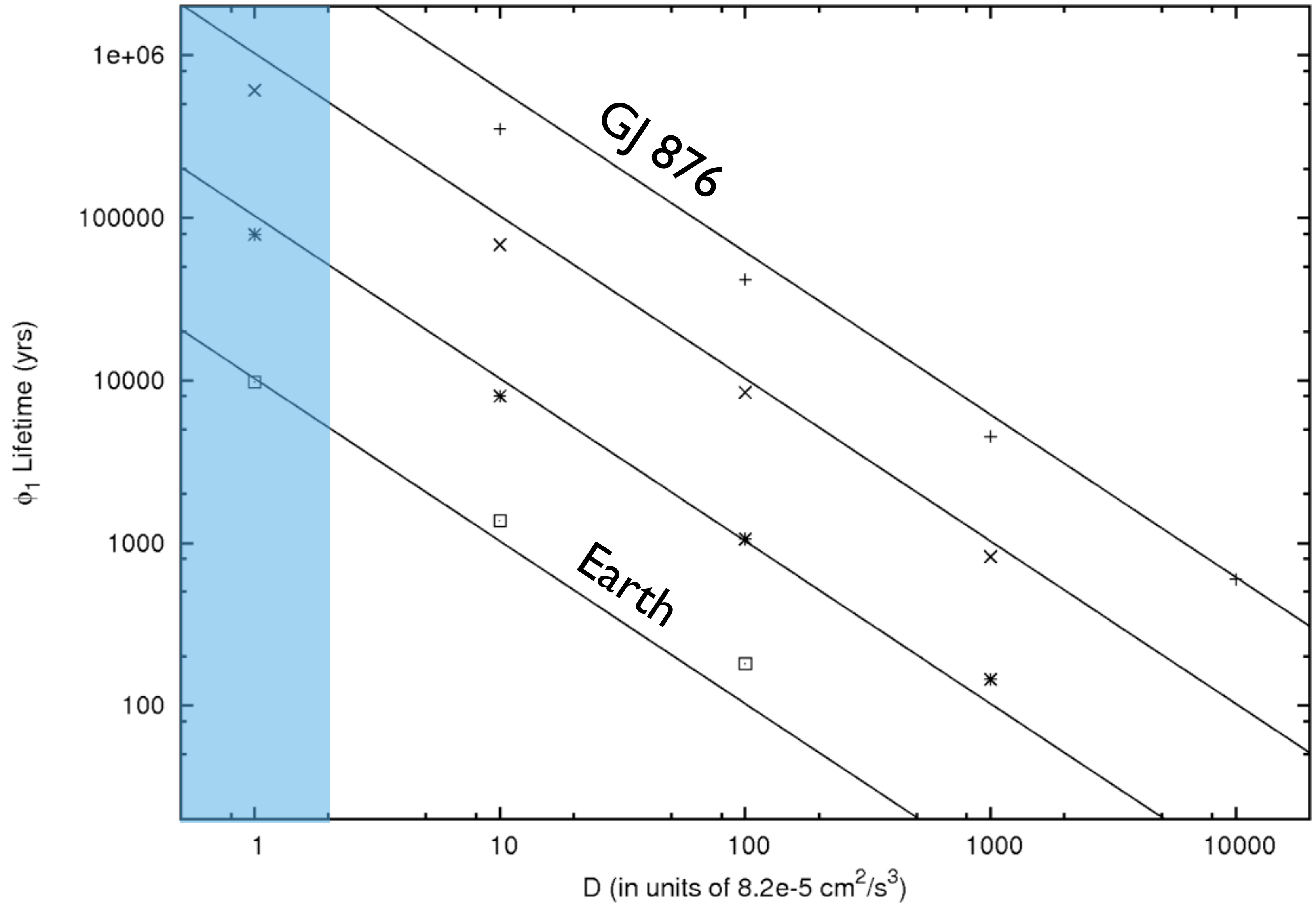
$$(\Delta\varpi)^2 = \frac{2.5}{e^2}\frac{\gamma Dt}{n^2a^2}$$

$$(\Delta e)^2 = 2.5\frac{\gamma Dt}{n^2a^2}$$



Rein & Papaloizou 2009, Rein 2010

Multi-planetary systems in mean motion resonance

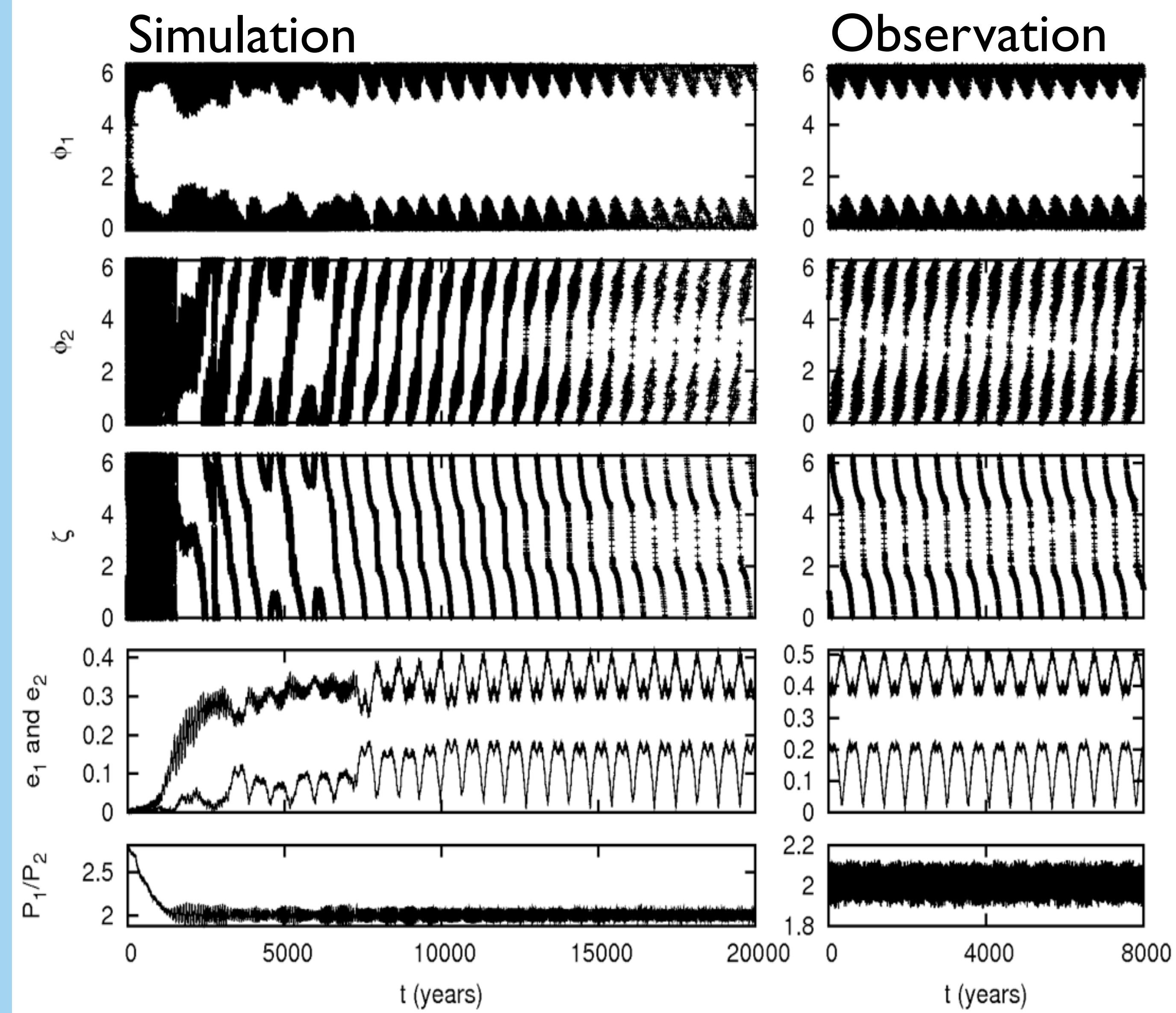


- Stability of multi-planetary systems depends strongly on diffusion coefficient
- Most planetary systems are stable

Rein & Papaloizou 2009

Modification of libration patterns

- HD128311 has a very peculiar libration pattern
- Can not be reproduced by convergent migration alone
- Turbulence can explain it
- More multi-planetary systems needed for statistical argument

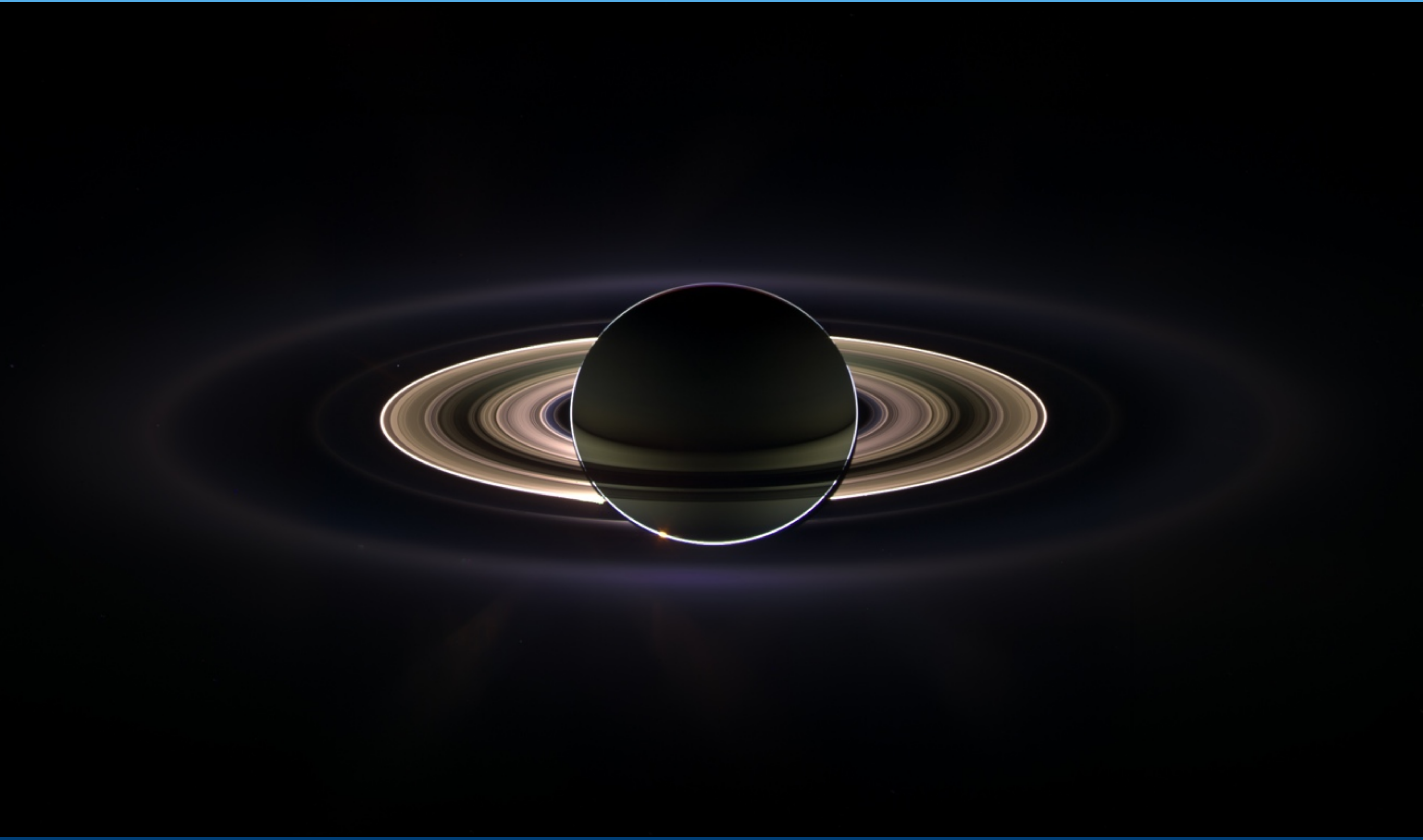


1. Multi-planetary systems

- Standard Model
- Turbulent disc

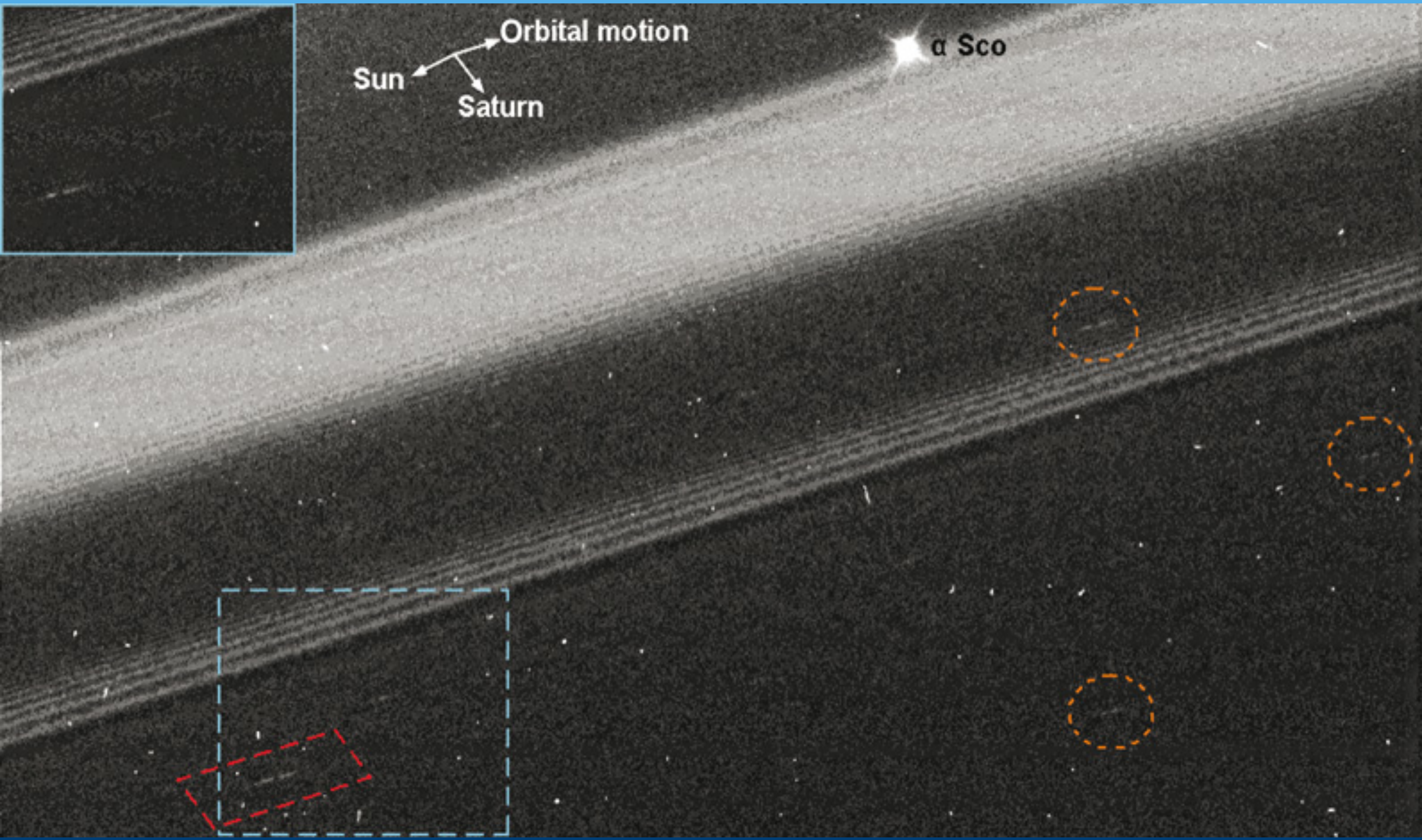
2. Saturn's rings

Cassini spacecraft



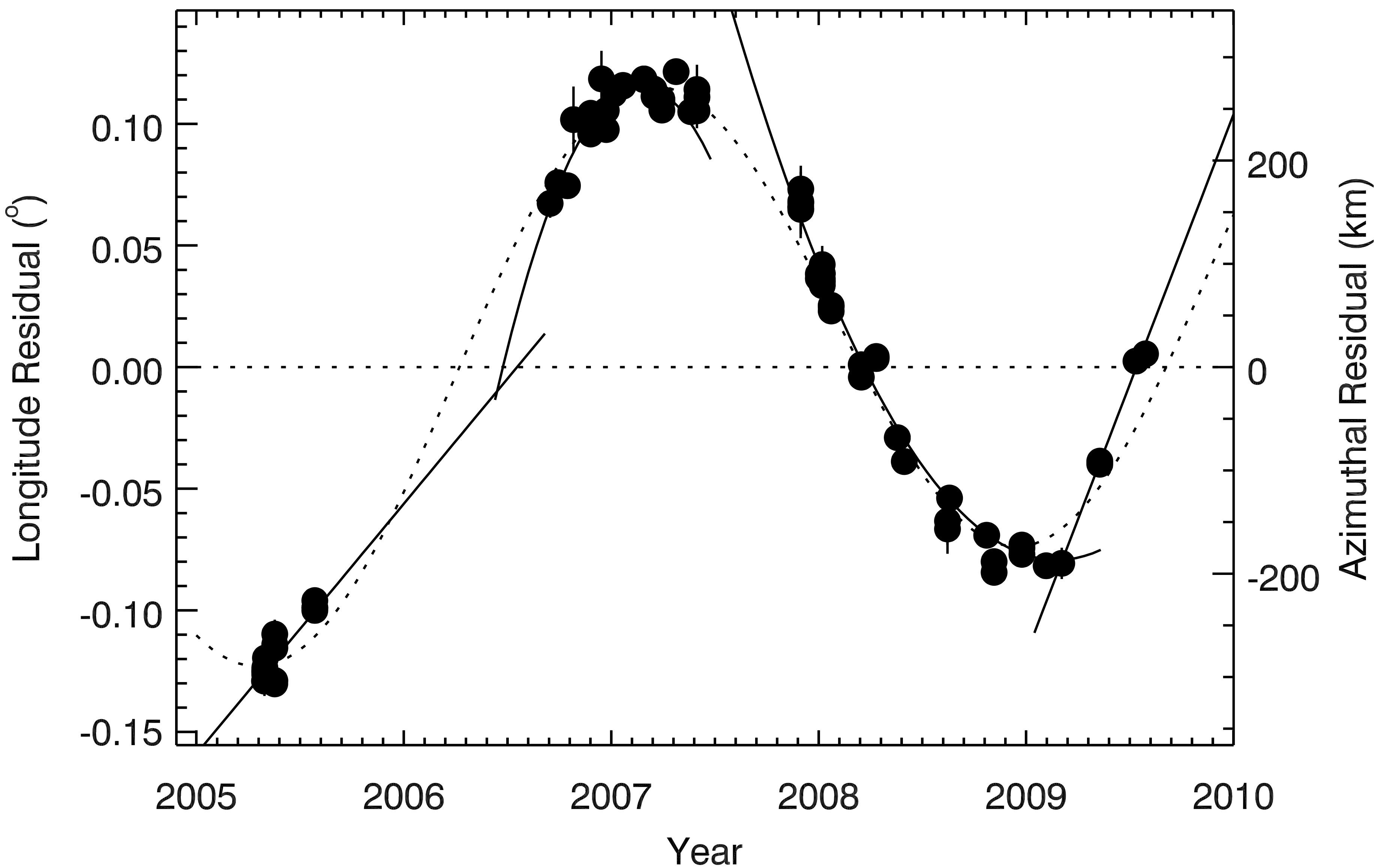
NASA/JPL/Space Science Institute

Propeller structures in A-ring



Porco et al. 2007, Sremcevic et al. 2007, Tiscareno et al. 2006

Observational evidence of non-Keplerian motion



Two different approaches

Analytic model

Describing evolution in a statistical manner

Rein & Papaloizou 2009, 2010

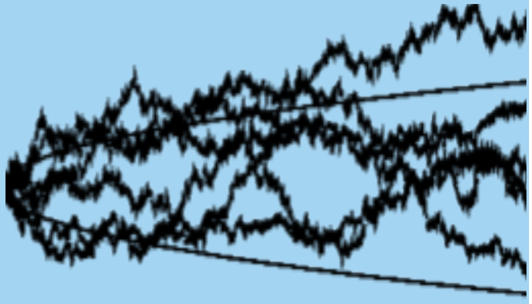
$$\Delta a = \sqrt{4\frac{Dt}{n^2}}$$

$$\Delta e = \sqrt{2.5\frac{\gamma Dt}{n^2 a^2}}$$

N-body simulations

Measuring random forces or integrating moonlet directly

Crida et al 2010, Rein & Papaloizou 2010



Effects contributing to the eccentricity evolution

- Laminar collisions
- Particles collisions
- Laminar horseshoe
- Laminar circulating
- Particles circulating
- Clumps circulating

→ **Equilibrium eccentricity**

Damping (green) | Excitation (red)

Rein & Papaloizou 2010, Crida et al 2010

... semi-major axis evolution

Particles collisions

Particles horseshoe

Particles circulating

Clumps circulating

Damping | Excitation

→

Random walk in semi-major axis

+Net "Type I" migration

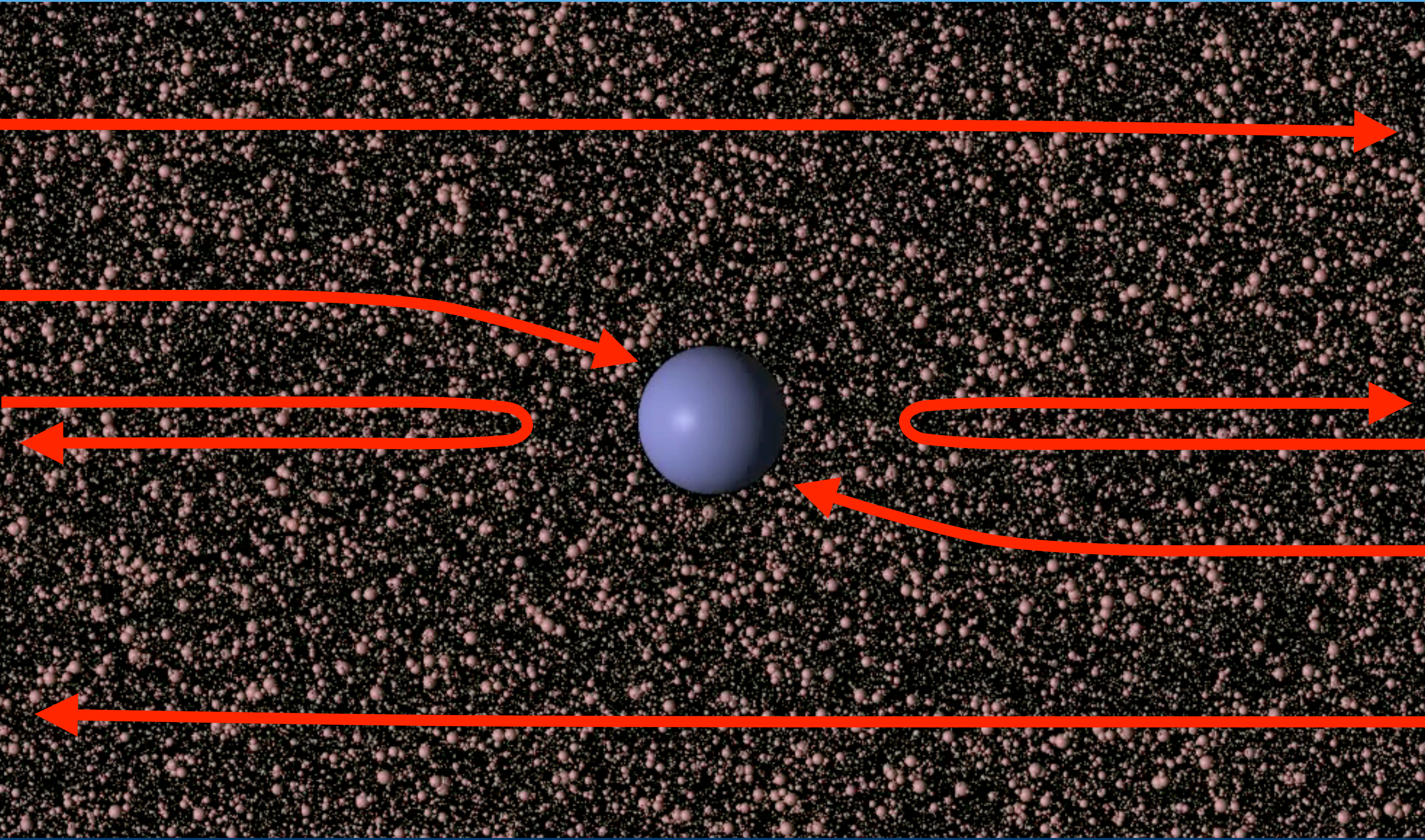
Rein & Papaloizou 2010, Crida et al 2010

GravTree

- 3D collisional N-body simulations
- Large N (~few million particles)
- Barnes-Hut tree used for gravity and collisions
- Parallelisation: pthreads and MPI
- Real-time visualisations with OpenGL

Rein, Lesur & Leinhardt 2010, Rein & Papaloizou 2010
Barnes and Hut 1986

Particle trajectories



Rein & Papaloizou 2010, Crida et al 2010

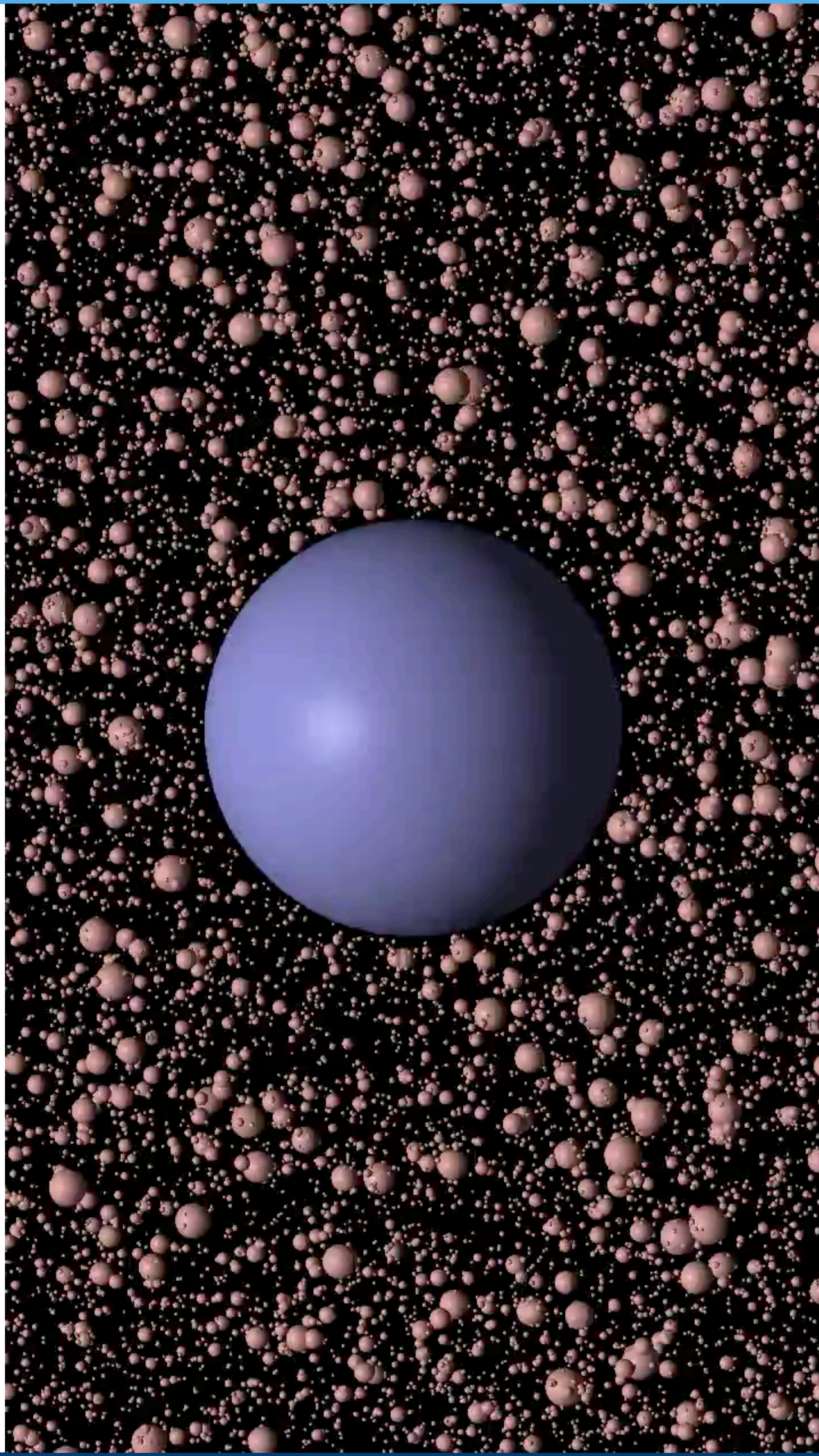
Results I: Moonlet is undergoing a random walk

Confirm analytic expression for mean eccentricity

Confirm analytic expression for random walk in semi-major axis

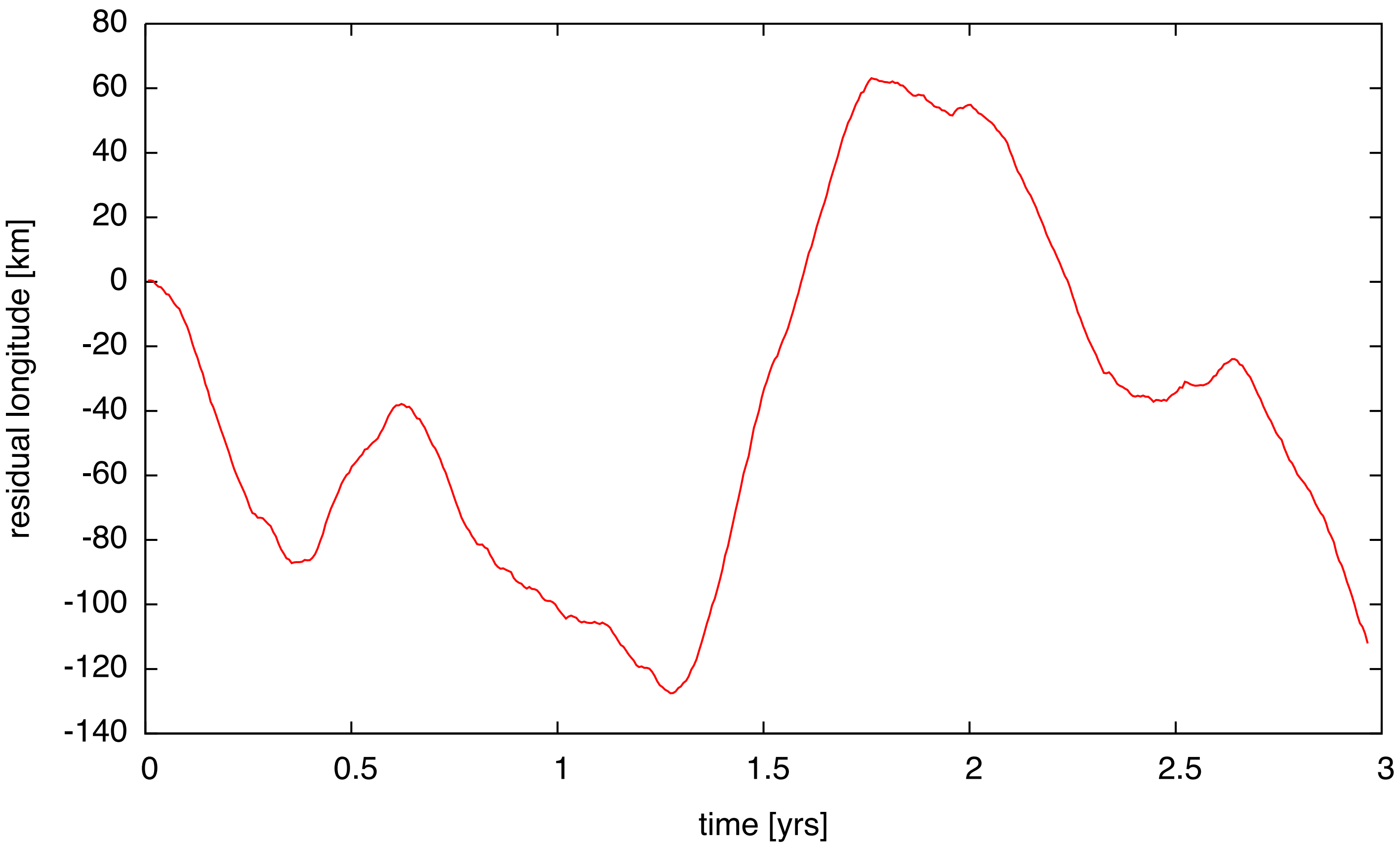
Identify most important effects

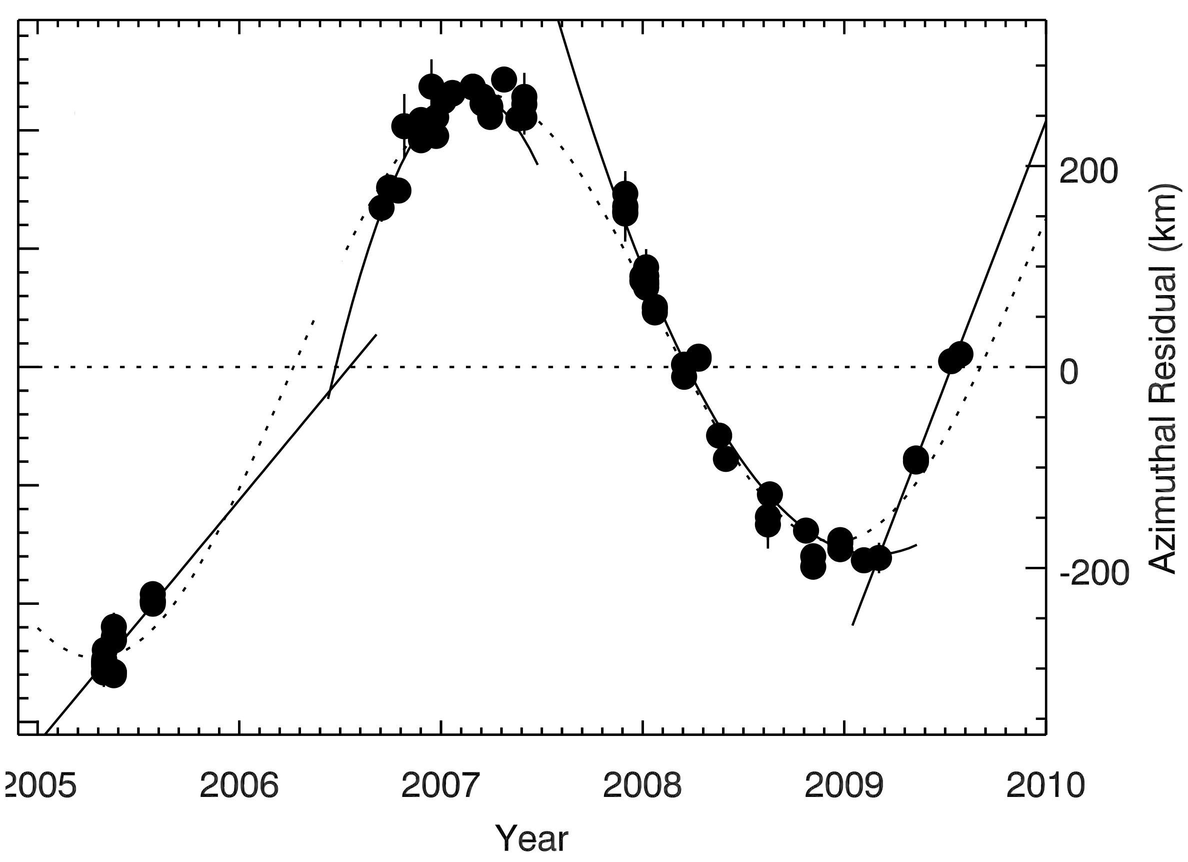
- Collisions (equipartition)
- Stochastic forces from circulating particles
- Stochastic forces from circulating clumps



Rein & Papaloizou 2010, Crida et al 2010

Results II: Comparison with observations





Conclusions

Conclusions

Multi-planetary systems and turbulence

- Multi-planetary system provide insight in otherwise unobservable formation phase
  - HD45364 formed in a massive disc
  - Turbulence can be traced by observing multi-planetary systems
  - HD 128311 has very peculiar libration pattern
  - Distinctive from non-turbulent migration scenarios
- Realistic MHD simulations will give a better estimate of diffusion coefficient
- More planetary systems allow a statistical argument

Moonlets in Saturn's rings

- Small scale version of the protoplanetary disc
- Dynamical evolution can be directly observed
  - Evolution is dominated by random-walk
  - Caused by collisions and gravitational wakes
- Might lead to independent age estimate of the ring system

Rein & Papaloizou 2009, 2010, Crida et al 2010

Thank you for your attention.